pn junction in equilibrium

day07

1. Review

The reading of Razavi section 2.2.1 was assigned and expected to have been completed by the start of this class session. Also §2.2.2, but your reading assignments are intentionally ahead of class time topics.

Tourbook table: Four currents in a semiconductor

2. Junctions

> This is also Tourbook: 2.6. Junctions §2.6.1. It is the Dr. White version of the Razavi book section of the same name.

The **goal** of these subsections is to develop a model of the *pn* junction that is useful for circuit design and analysis. We will end up with a relationship between current and voltage, and another expression for the junction's capacitance (charge storage).

The map for the next three sessions is:

1. equilibrium (open-circuit)
2. reverse bias (capacitance, leakage)
3. forward bias (I/V curve)

2.1. *pn* junction in equilibrium

Imagine taking two separate bars of silicon, one doped to be *n*-type (N_D) and the other doped to be *p*-type (N_A). Push these bars together so that there is a single bar where the middle abruptly changes doping types and levels.

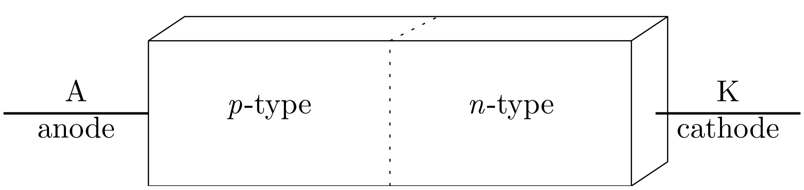

Figure 1. Semiconductor bar doped half p-type and half n-type

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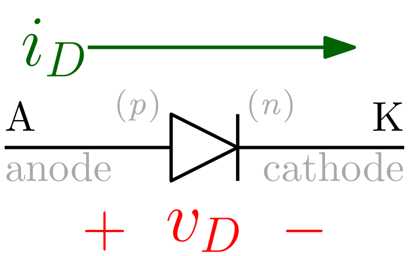

Figure 2. Diode symbol and labels

At the moment of contact, there is a huge concentration **gradient** for both holes and electrons. The *p*-type region with a majority of holes is right next to the *n*-type region with very few holes and vise versa.

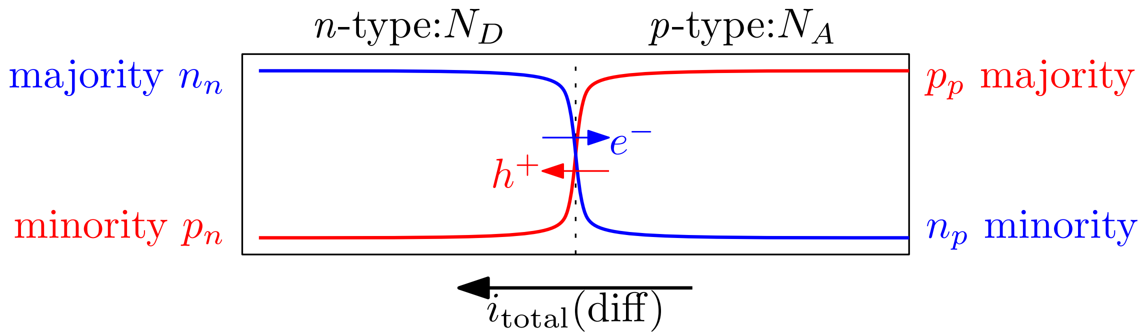

Figure 3. Initial diffusion currents at the junction.

Draw the diode symbol in the same orientation as Figure 3 below:

The *n* and *p* sides are reversed compared to previous figure. This is to match our other reference materials.

▸ What happens?

After a long time, (with $\exp(-t/\tau)$ behavior, this diffusion will eventually stop once the electron and hole densities have become uniform:

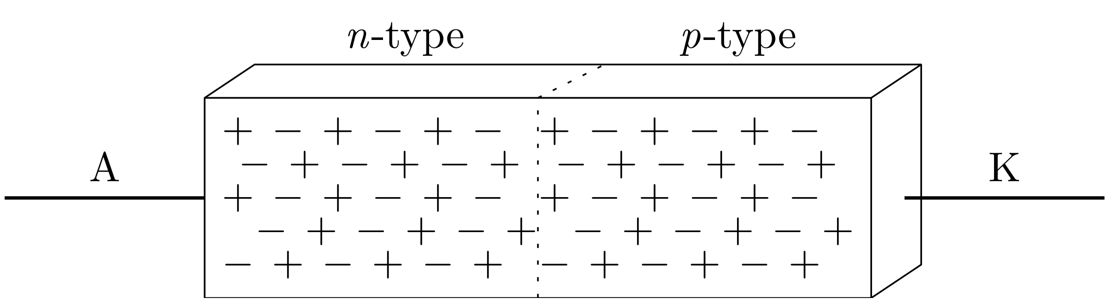

Figure 4. Equilibrium once diffusion has ceased

Take a minute to explain to someone next to you why this might initially make sense, but, in fact, cannot possibly be true.

NO !!! We have forgotten about the **protons** that are in this material!

> *If your protons are moving, you have bigger problems.*

Think about the Group IV dopant atom (say phosphorus) on the *n* side of the junction. That extra electron is in a region of many free electrons and next to a region with few. So it will tend to move (diffuse) to the right towards the *p* region.

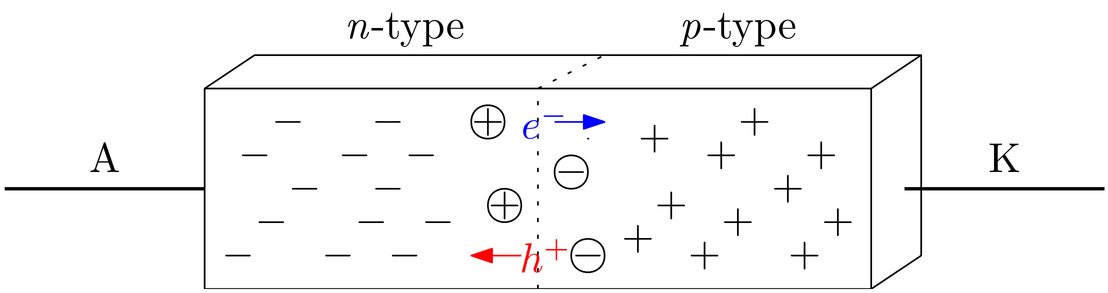

Figure 5. Fixed ions in the transition region

This particular free electron came from the 5th outer valence electron of the phosphorus dopant atom. Therefore, when it diffuses away, the 15th proton in the nucleus is left without a charge-balancing mate. We now have a +1 positively-charged phosphorus **ion**.

Similarly, when the hole contributed by the Group III dopant (say boron) on the *p* side diffuses to the left, we remember that the hole was filled with an electron. This electron fills the 4th outer valence "slot" and completes the covalent bonding structure in that area. Don't forget that boron is short a proton that matched the now-filled hole. Therefore the region has a net −1 negatively-charged boron **ion**.

Both of these types of ions *cannot move* because the charges are either a) part of the nucleus, or b) a **bound** electron. This means that we have:

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called the **space charge region** or the **depletion region** because the volume is eventually *depleted* of free charge carriers.

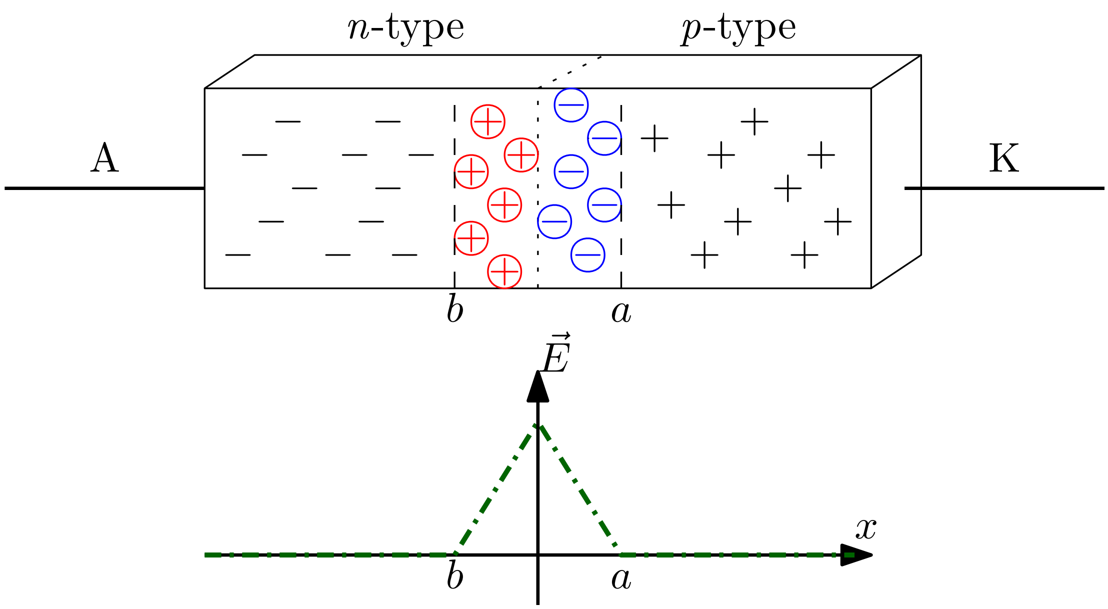


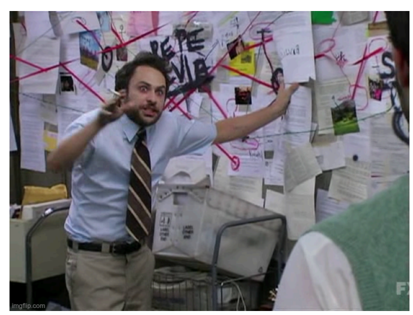
Figure 6. E-field inside of a pn junction

Now that there are separated charges, Gauss's law, $Q = \nabla \bullet \left(\epsilon \vec{E}\right)$, implies that (forces?) there is a non-zero E-field inside the depletion region. This field is maximum at the boundary between the *n* and *p* type doping and, for this example, points to the right.

Recall that the **diffusion** currents are to the *left* and the drift current from the E-field is opposite, to the *right*. Because this semiconductor bar is not connected in a circuit, **KCL** forces the current at every *x* location to be zero. The junction is in **equilibrium** and the drift and diffusion currents must be equal and opposite.

$$|J_p(\text{drift}) + J_n(\text{drift})| = |J_p(\text{diff}) + J_n(\text{diff})| \tag{1}$$

Take another minute to explain to someone who is nice enough to listen why Equation (1) violates all manner of rules:



> **NO !** (again). The currents must be balanced for holes and electrons **individually**, or it makes no physical sense.

$$J_p(\text{drift}) + J_p(\text{diff}) = 0 \tag{2}$$
$$J_n(\text{drift}) + J_n(\text{diff}) = 0 \tag{3}$$

Take the free electrons equation and substitute their definitions:

$$q\mu_n n\vec{E} = -qD_n\frac{dn}{dx} \tag{4}$$
$$-\mu_n n\frac{dV}{dx} = -D_n\frac{dn}{dx} \tag{5}$$

Integrate and divide both sides by n

$$\mu_n \int_b^a dV = -D_n \int_{n_p}^{n_n} \frac{dn}{n} \tag{6}$$
$$V(a) - V(b) = -\frac{D_n}{\mu_n}\ln\left(\frac{n_p}{n_n}\right) \tag{7}$$

The left side is the potential difference developed across the depletion region and is given the symbol V_0. Also, recall Einstein's relation, $\frac{D}{\mu} = \frac{k_B T}{q}$, and substitute

$$V_0 = \frac{k_B T}{q}\ln\left(\frac{n_n}{n_p}\right) \tag{8}$$

We usually are only concerned with the *magnitude* of V_0; notice that swapping the fraction so the majority carriers term (n_n) is in the numerator cancels the leading negative sign. Recognize, for the next section, that the **higher** potential is on the *n* side, at location *b*, given the E-field direction. A similar process can be done to find V_0 from the hole currents.

One final set of substitutions using our (good) approximations of majority and minority carrier concentrations

$$n_n \to N_D \tag{9}$$
$$n_p \to \frac{N_A}{n_i^2} \tag{10}$$

$$V_0 = \frac{k_B T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) \tag{11}$$
$$= V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) \tag{12}$$

Memorize this

$V_T = \dfrac{k_B T}{q}$ is called the **thermal voltage**.

- It is 25.85 mV at 300 K (26.85°C)

Both 25 mV and 26 mV are used in practice for hand calculations at "room temperature." You will also see 40 in some equations ($= 1/25\mathrm{mV}$). Because V_T is usually inside an exponent, the numerical result of a calculation can be sensitive to which approximation you used.

Therefore it is best to *state* your assumptions instead of assuming the reader rounds the same direction as you do.

3. Problems for pairs

- Memorize the entirety of Figure 2 and Figure 7. It relates names of each doped side, voltage and current signs, and symbols. This figure's content in all aspects will be on every exam from now forward.

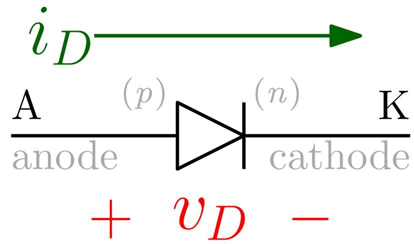


Figure 7. Tourbook Figure 8

3.1. Build your potential

The book derives the **built-in potential** V_0 from book equation (2.62).

- [ ] Do the similar derivation, but begin with the electron current balance between drift and diffusion action from book equation (2.63). You will get a different answer than book equation (2.68)!

3.2. This field is conservative

Look back at book **Figure 2.18**. There are two regions of the semiconductor bar that suddenly change their doping types and levels at the middle dotted line.

Focus on the curve for one type of charge carriers, e.g. holes. The concentration starts low on the left since this is the (majority-)n-type region, then then increases until it is the majority type of carrier in the right side.

Now, go back to the calculus used to derive the **built-in potential**. This needs the E-field (or voltage) versus position (x, here), and needs charge carrier concentration.

- Why doesn't the exact shape of either the electric field or the hole/electron concentration make a difference?

4. Symbology

Write in words what each of these symbols mean:

- n_n
- p_p
- n_p
- p_n

Draw a figure of two opposite-doped parts of a bar of silicon that also shows these symbols. Also include where N_A and N_D would show up.

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1. (not) The Muppets