

ECE 340 - Solving non-linear equations

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The *guess-and-iterate* method

1. Forward-biased diode circuit

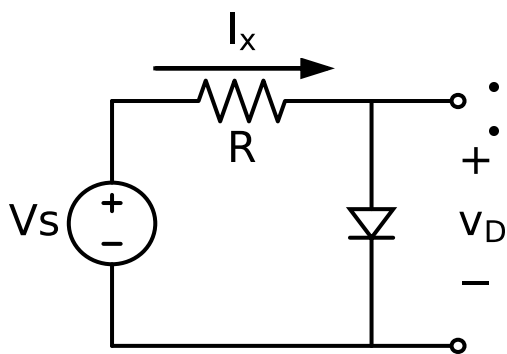


Figure 1. Simplest circuit with a diode

Your first diode circuit problem! Find I_x .

- Write down a value for R :
- Write down a value for V_s :

Do a little bit of (symbolic) circuit analysis:

$$I_x = \frac{V_s - v_D}{R} \quad (1)$$

$$I_x = i_D = I_S \exp\left(\frac{v_D}{V_T}\right) \quad (2)$$

Ohm's law, KCL, and "diode's law" relating current and voltage in a diode are all we need, so we end up with one equation with one unknown:

$$\frac{V_s - v_D}{R} = I_S \exp\left(\frac{v_D}{V_T}\right) \quad (3)$$

Easy enough to crank the algebra handle and solve for $v_D = \dots$

Except you find you can't do it. Ever.

This equation is *transcendental* and does not have an analytical (symbolic) solution. Build this circuit in the lab and you will prove that the circuit does indeed have a solution — because you can **measure** it. Despite having no **symbolic** solution, it is possible to find a numerical solution by *searching*. Mathematics folk call these techniques **numerical methods**. My personal favorite is **bisection** because the *idea* even works as a solid troubleshooting strategy.

Before we get to the calculator-only technique, first consider the two device equations: resistor (1) and diode (2). Draw the I_x vs. v_D plot for each equation separately and recognize that the location where they intersect has precisely the same meaning as the solution to equation (3).

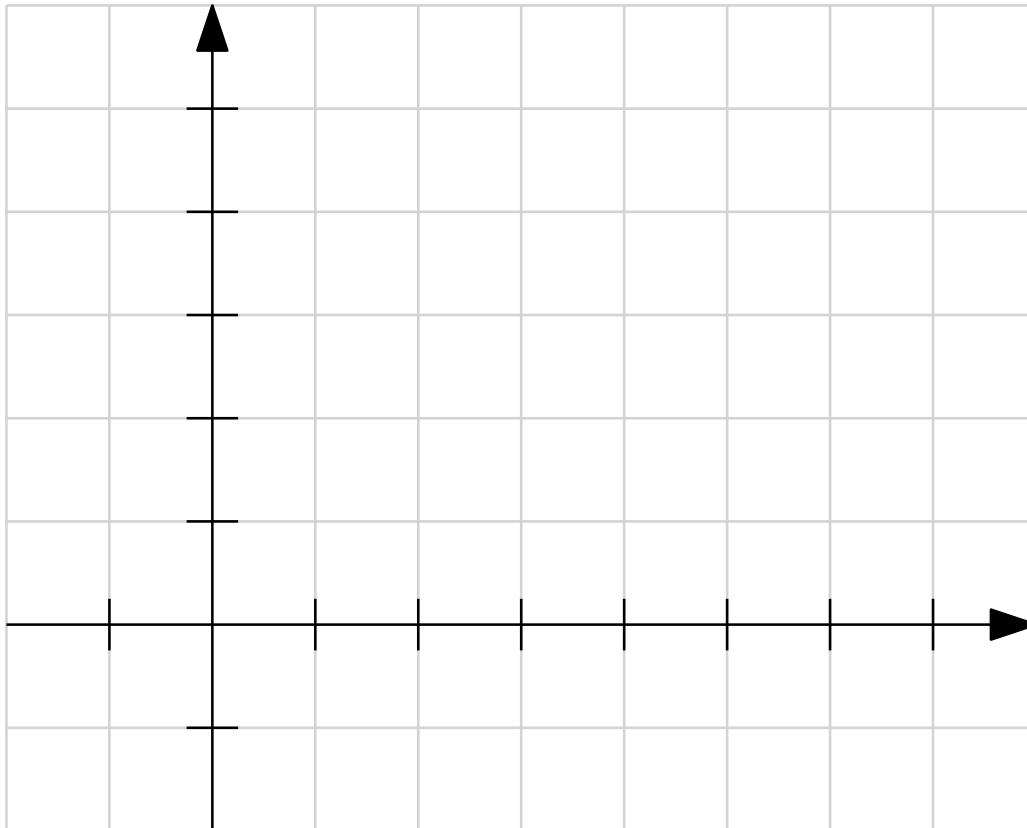


Figure 2. Load line plot, resistor and diode currents.

2. Guess and iterate technique

2.1. Setup

Begin this technique by finding two separate equations which include the same two unknowns.. In this example, we have (1) and (2) which:

- are equal to each other by KCL and
- both depend on the same parameter v_D .

Next, *algebra*^[1] the equations to solve one for the first variable and the other for the second variable. For the example, keep equation (1) as-is and solve (2) for v_D :

$$v_D = V_T \ln\left(\frac{I_x}{I_S}\right) \quad (4)$$

2.2. Guess

Choose a starting value for one of the variables by guessing, engineering style.^[2] For example, it should be obvious from the schematic that I_x can't be negative, nor can it be larger than the value if the diode was a short-circuit: $I_x \in \left[0, \frac{V_s}{R}\right]$. So, choose a number in the middle.

Don't think so hard! Just choose the middle value of the I_x range and proceed.

2.3. Iterate

Take this I_x and plug into equation (4) to get a value for v_D . Take this value of v_D , plug into equation (1) to get an *updated* value for I_x . Plug this new I_x back into equation (4).

Continue substituting one result into the other equation until you obtain enough stable digits for your required precision. It is common to only need **3 digits** of precision, which represents less than 1% difference between the true value; if using this number in another calculation, use **4 digits** instead.

3. Practice

For Figure 1, use 3.3 V, 470 Ω , and a diode with $V_T=26$ mV, $\eta=1.75$, and $I_S=1$ nA. Use the *guess-and-iterate* method to find the circuit solution for v_D and I_x .

See the in-class calculator demonstration for how to reduce the work of computing each step of the iteration.

Step	v_D	I_x
1		
2		
3		
4		
5		
6		
7		

4. Reference

https://agnd.net/valpo/340/notes_guess-iterate.html has even more notes on this fast method, including **animations**.

1. the verb form

2. Scientific wild-ass guess , Estimating Software Projects: The art of the SWAG is also good.